Mat 2233
Chapter 6 & 7
practice solutions

① marginal cost $\frac{dc}{dx} = 2x - 6$

\$50 to produce 10 units

find $C(x)$

$$\int (2x-6)\,dx = \frac{2x^2}{2} - 6x + c$$

$$C(x) = x^2 - 6x + c$$

$$C(10) = (10)^2 - 6(10) + c = 50$$

$$100 - 60 + c = 50$$

$$40 + c = 50$$

$$c = 10$$

$$\boxed{C(x) = x^2 - 6x + 10}$$ (A)

② $\int (e^x + x^e)\,dx$

$$\int e^x\,dx + \int x^e\,dx$$

$$\boxed{e^x + \frac{x^{e+1}}{e+1} + c}$$ (B)

③ $\int (4e^{-2x} + x^{-1} - x^{-2})\,dx$

$$\frac{4}{-2}e^{-2x} + \ln|x| - \frac{x^{-1}}{-1} + c$$

$$= \boxed{-2e^{-2x} + \ln|x| + \frac{1}{x} + c}$$

④ $\int \frac{1}{8x}\,dx$

$$= \frac{1}{8}\int \frac{1}{x}\,dx$$

$$= \boxed{\frac{1}{8}\ln|x| + c}$$

⑤ $\int (1-x^3)^4 x^2\,dx$

$u = 1 - x^3$ (D)

⑥ $\int \frac{x}{\sqrt{1+2x^2}}\,dx$

$u = 1 + 2x^2$ (A)

⑦ $\int \frac{(\ln x)^6}{x}\,dx$

$$u = \ln x$$

$$du = \frac{1}{x}\,dx$$

$$\int u^6\,du$$

$$\frac{u^7}{7} + c$$

(C) $\boxed{\frac{1}{7}(\ln x)^7 + c}$

Answer not among the choices.

(8) $\sum_{j=0}^{3} 2^j - 1 = (2^0-1) + (2^1-1) + (2^2-1) + (2^3-1)$

$= (1-1) + (2-1) + (4-1) + (8-1)$

$= 0 + 1 + 3 + 7 = \boxed{11}$ A

(9) $\int_1^3 x^2\,dx$ left Riemann sum with $n=4$ (start with left endpoint)

$\sum_{k=0}^{n-1} f(x_k)\,\Delta x \qquad \Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$

$\left[f(1) + f\left(\tfrac{3}{2}\right) + f(2) + f\left(\tfrac{5}{2}\right)\right]\left(\tfrac{1}{2}\right)$ B

$f(1) = (1)^2 = 1$

$f\left(\tfrac{3}{2}\right) = \left(\tfrac{3}{2}\right)^2 = \frac{9}{4}$

$f(2) = (2)^2 = 4$

$f\left(\tfrac{5}{2}\right) = \left(\tfrac{5}{2}\right)^2 = \frac{25}{4}$

$\left(1 + \frac{9}{4} + 4 + \frac{25}{4}\right)\left(\frac{1}{2}\right)$

$\left(\frac{4}{4} + \frac{9}{4} + \frac{16}{4} + \frac{25}{4}\right)\left(\frac{1}{2}\right)$

$\left(\frac{53}{4}\right)\left(\frac{1}{2}\right) = \boxed{\frac{54}{8}} = \boxed{6.75}$

(10) $\int_1^2 \frac{\ln x}{x}\,dx = \int_{x=1}^{x=2} u\,du = \frac{u^2}{2}\Big|_{x=1}^{x=2} = \frac{(\ln x)^2}{2}\Big|_1^2$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \frac{(\ln 2)^2}{2} - \frac{(\ln 1)^2}{2} = \boxed{\frac{1}{2}(\ln 2)^2}$

A

(11)
$$\int_{-5}^{4} \sqrt{4-x}\,dx = \int_{-5}^{4} (4-x)^{1/2}\,dx \qquad u = 4-x \quad du = -dx$$

$$-\int_{x=-5}^{x=4} u^{1/2}\,du = -\frac{2}{3}u^{3/2}\Big|_{x=-5}^{x=4} = -\frac{2}{3}(4-x)^{3/2}\Big|_{-5}^{4}$$

$$= -\frac{2}{3}(4-4)^{3/2} - \left[-\frac{2}{3}(4--5)^{3/2}\right] \qquad \text{(A)}$$

$$= -\frac{2}{3}(0)^{3/2} + \frac{2}{3}(9)^{3/2} = \frac{2}{3}(27) = \boxed{18}$$

(12)
$$\int 24x(x-5)^2\,dx \qquad \text{by parts}$$

$$u = 24x \qquad dv = (x-5)^2\,dx$$

$$du = 24\,dx \qquad v = \int (x-5)^2\,dx = \frac{(x-5)^3}{3}$$

$$\int u\,dv = uv - \int v\,du$$

$$= (24x)\left(\frac{x-5}{3}\right)^3 - \int \frac{(x-5)^3}{3}(24\,dx)$$

$$= 8x(x-5)^3 - 8\int (x-5)^3\,dx \qquad \text{(C)}$$

$$= 8x(x-5)^3 - \frac{8(x-5)^4}{4} + C$$

$$= \boxed{8x(x-5)^3 - 2(x-5)^4 + C}$$

* note * the actual C has $(x-5)^2$, it should be $(x-5)^4$

(13) $\int e^x(x+2)\,dx$ integration By parts (with shortcut)

$u = (x+2)$ $dv = e^x\,dx$

	derivative	integral
+	$x+2$	e^x
$-\int$	1	e^x

$(x+2)e^x - \int 1\cdot e^x\,dx = \boxed{(x+2)e^x - e^x + C}$

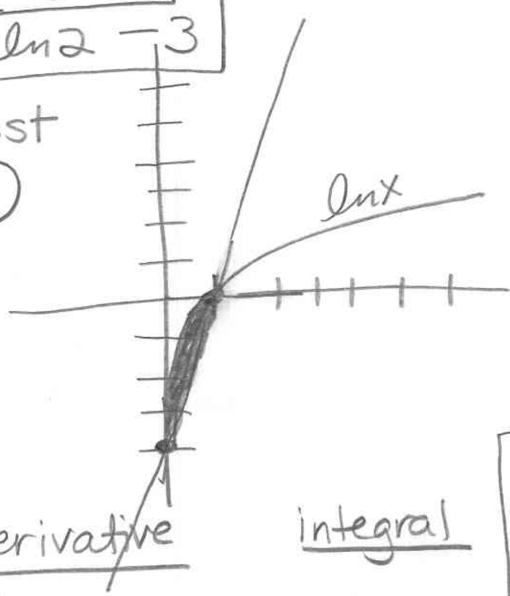
answer is close to (B)

(14) $y = \ln x$ and $y = 4x - 4$ from 1 to 2

Result

$2\ln 2 - 3$

Almost (A)



$$\int_1^2 (\ln x - (4x-4))\,dx = \int_1^2 (\ln x - 4x + 4)\,dx$$

$\ln x$ must be integrated by parts.

	derivative	integral
+	$\ln x$	1
$-\int$	$\frac{1}{x}$	x

$x\ln x - \int \underbrace{\frac{1}{x}\cdot x\,dx}_{dx}$

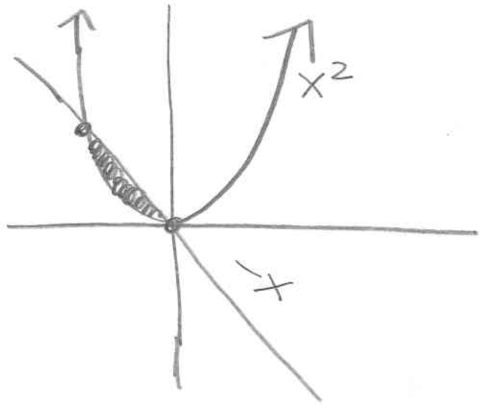
$$\int \ln x\,dx = x\ln x - x$$

$$\int_1^2 (\ln x - 4x + 4)\,dx$$

$$= x\ln x - x - \frac{4x^2}{2} + 4x \Big|_1^2$$

$$= 2\ln 2 - 2 - 2(2)^2 + 4(2) - (1\ln 1 - 1 - 2(1)^2 + 4(1))$$

(15) Area between curves $y=x^2$ and $y=-x$



$$\int_{-1}^{0} (-x)-x^2\,dx$$

$$= \int_{-1}^{0} (-x-x^2)\,dx$$

$$= -\frac{x^2}{2} - \frac{x^3}{3}\Big|_{-1}^{0}$$

find intersection pts

$x^2 = -x$

$x^2 + x = 0$

$x(x+1) = 0$

$x=0 \qquad x+1=0$

$\qquad\qquad x=-1$

$$= \left[-\frac{(0)^2}{2} - \frac{(0)^3}{3}\right] - \left[-\frac{(-1)^2}{2} - \frac{(-1)^3}{3}\right]$$

$$= 0 - \left[-\frac{1}{2} + \frac{1}{3}\right]$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \boxed{\frac{1}{6}}$$

(16) Average

$f(x) = \sqrt{x} \quad [1, 9]$

$$\bar{A}(x) = \frac{1}{9-1}\int_{1}^{9} \sqrt{x}\,dx = \frac{1}{8}\int_{1}^{9} x^{1/2}\,dx$$ (D)

$$= \frac{1}{\cancel{8}_4} \cdot \frac{\cancel{2}}{3} x^{3/2}\Big|_{1}^{9} = \frac{1}{12}(9)^{3/2} - \frac{1}{12}(1)^{3/2}$$

$$= \frac{1}{12}(27) - \frac{1}{12}(1) = \frac{26}{12} = \boxed{\frac{13}{6}}$$

(17) 3 unit moving average

$f(x) = x^2$

$$\bar{f}(x) = \frac{1}{n}\int_{x-n}^{x} f(t)\,dt$$

$(x-3)^3$

$(x-3)(x^2-6x+9)$

$$\begin{array}{r} x^3-6x^2+9x \\ -3x^2+18x-27 \\ \hline x^3-9x^2+27x-27 \end{array}$$

$$\bar{f}(x) = \frac{1}{3}\int_{x-3}^{x} t^2\,dt = \frac{1}{3}\frac{t^3}{3}\Big|_{x-3}^{x} = \boxed{\frac{x^3}{9} - \frac{(x-3)^3}{9}}$$

$$= \frac{x^3}{9} - \left[\frac{x^3-9x^2+27x-27}{9}\right]$$

$$= \cancel{\frac{x^3}{9}} - \cancel{\frac{x^3}{9}} + \frac{9x^2}{9} - \frac{27x}{9} - \frac{27}{9}$$

$$= \boxed{x^2-3x-3}$$

B

(18) \$5000 compounded continuously $(A = Pe^{rt})$

8% 5 yrs

C

Average money over 5 years

$A = 5000e^{0.08t}$

$$\text{average} = \frac{1}{5-0}\int_0^5 5000e^{0.08t}\,dt$$

$$= \frac{1}{5}\left[\frac{5000}{0.08}e^{0.08t}\Big|_0^5\right.$$

$= 18,647.81 - 12,500 = \boxed{\$6147.81}$

Skip 19 & 20

Bonus Problems

① $\int_0^1 \frac{x^3}{\sqrt{x^2+1}}\,dx$ parts

	derivative	Integrate
+	x^2	$\frac{x}{\sqrt{x^2+1}}$
$-\int$	$2x$	$\frac{1}{2}\cdot\frac{2}{1}(x^2+1)^{1/2}$
		$(x^2+1)^{1/2}$

$= x^2\cdot(x^2+1)^{1/2} - \int 2x\,(x^2+1)^{1/2}\,dx$

$= x^2(x^2+1)^{1/2} - \frac{2}{3}(x^2+1)^{3/2}\Big|_0^1$

$= \left[(1)^2(1^2+1)^{1/2} - \frac{2}{3}(1^2+1)^{3/2}\right] - \left[0(0+1)^{1/2} - \frac{2}{3}(0+1)^{3/2}\right]$

$= \left[\sqrt{2} - \frac{2}{3}(2)^{3/2}\right] - \left[-\frac{2}{3}(1)^{3/2}\right]$

$= 1\sqrt{2} - \frac{2}{3}\cdot 2\sqrt{2} + \frac{2}{3} = \frac{2}{3} + \frac{3}{3}\sqrt{2} - \frac{4}{3}\sqrt{2}$

$= \boxed{\frac{2}{3} - \frac{1}{3}\sqrt{2}}$

② $\int x^2 \ln 4x\,dx$ parts

	derivative	integral
+	$\ln 4x$	x^2
$-\int$	$\frac{1}{4x}\cdot 4$	$\frac{1}{3}x^3$

$= \frac{1}{3}x^3 \ln 4x - \int \frac{1}{3}x^2\,dx = \boxed{\frac{1}{3}x^3 \ln 4x - \frac{1}{9}x^3 + C}$

Differs from soln but is correct.

Check for 2

$$\boxed{\frac{1}{3}x^3 \ln 4x - \frac{1}{9}x^3 + c}$$

take it's derivative

$$\frac{1}{3}x^3 \cdot \frac{1}{\cancel{4}x} \cdot \cancel{4} + \frac{1}{\cancel{3}} \cdot \cancel{3}x^2 \ln 4x - \frac{1}{9} \cdot 3x^2 + 0$$

$$\cancel{\frac{1}{3}x^2} + x^2 \ln 4x - \cancel{\frac{1}{3}x^2}$$

3 $$\boxed{\int \frac{(\sqrt{x-1} - 2)^3}{\sqrt{x-1}}\, dx}$$

$$u = \sqrt{x-1} - 2$$

$$du = \frac{1}{2}(x-1)^{-1/2}\, dx$$

$$du = \frac{1}{2\sqrt{x-1}}\, dx$$

$$2\, du = \frac{1}{\sqrt{x-1}}\, dx$$

$$2\int u^3\, du$$

$$= \cancel{2} \cdot \frac{u^4}{\cancel{4}_{\,2}} + c$$

$$= \boxed{\frac{(\sqrt{x-1} - 2)^4}{2} + c}$$